

Lecture 13 Slides

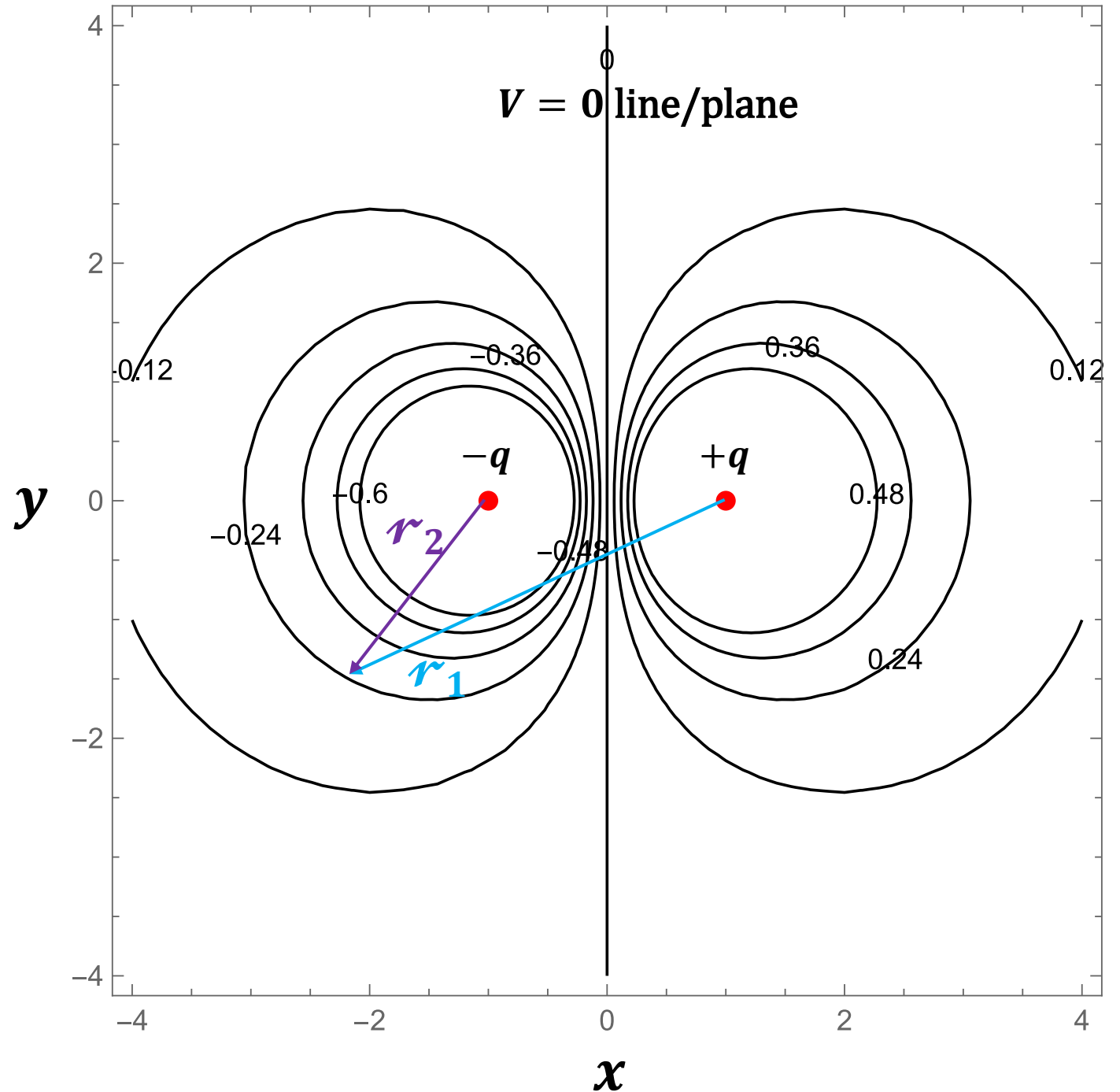
Phys 313 Fall 2026

Equipotential Lines

due to 2 point charges

$$V(x, y) = \frac{q}{4\pi\epsilon_0 r_1} - \frac{q}{4\pi\epsilon_0 r_2}$$

$$V(x, y) = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{(x-1)^2 + y^2 + z^2}} - \frac{1}{\sqrt{(x+1)^2 + y^2 + z^2}} \right)$$



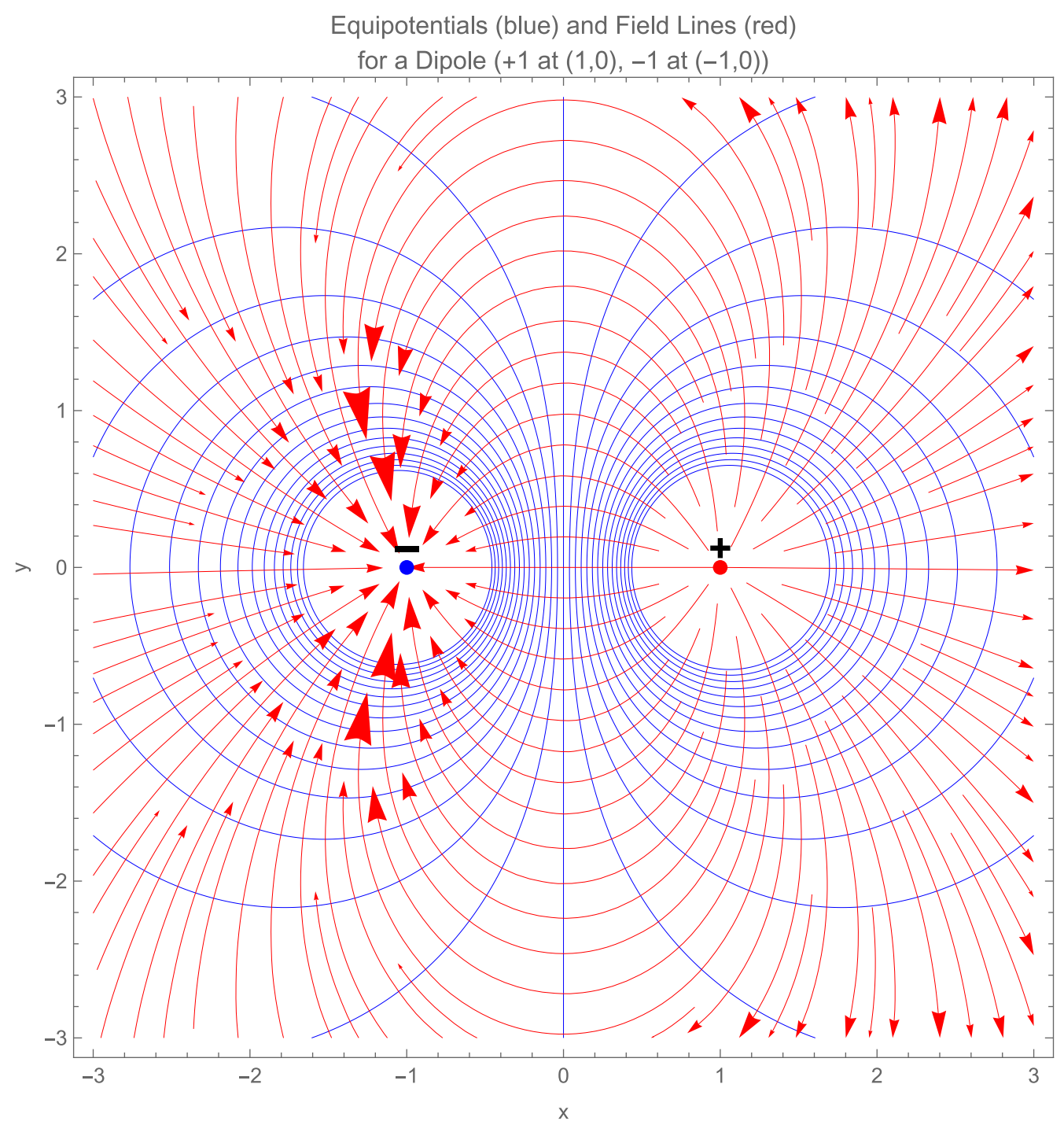
Mathematica Code to Generate Equipotential Lines for 2 point charges

```
V[x_, y_] := 1 / Sqrt[(x - 1)^2 + y^2] - 1 / Sqrt[(x + 1)^2 + y^2]
plot1 = ContourPlot[V[x, y], {x, -4, 4}, {y, -4, 4}, ContourShading → None, ContourLabels → True, Contours → 10];

plot2 = dotCoordinates = {{1, 0}, {-1, 0}};
plot2 = ListPlot[dotCoordinates, PlotStyle → {PointSize[0.02], Red}];

Show[plot1, plot2]
```

Equipotential Lines and Electric Field Lines 2 point charges



```
(*Two-charge system:+1 at (1,0),-1 at (-1,0)*)Clear[V, Ex, Ey, x, y]
```

```
q1 = 1; pos1 = {1, 0};
```

```
q2 = -1; pos2 = {-1, 0};
```

```
(*Electric potential,regularized to avoid singularities*)
```

```
V[x_, y_] := q1 / Sqrt[(x - pos1[[1]])^2 + (y - pos1[[2]])^2 + 0.01] + q2 / Sqrt[(x - pos2[[1]])^2 + (y - pos2[[2]])^2 + 0.01]
```

```
(*KEY FIX:use=(Set),not:=(SetDelayed),so D is evaluated ONCE symbolically at definition time,  
producing explicit formulas.This avoids D ever being called with numeric arguments later.*)
```

```
Ex[x_, y_] = -D[V[x, y], x];
```

```
Ey[x_, y_] = -D[V[x, y], y];
```

```
range = 3;
```

```
equipotentials = ContourPlot[V[x, y], {x, -range, range}, {y, -range, range}, Contours → 30, ContourShading → None,  
ContourStyle → Directive[Blue, Thin], PlotPoints → 80, MaxRecursion → 3];
```

```
fieldLines = StreamPlot[{Ex[x, y], Ey[x, y]}, {x, -range, range}, {y, -range, range}, StreamPoints → Fine, StreamColorFunction → None,  
StreamStyle → Directive[Red, Thin], StreamScale → Full];
```

```
chargeMarkers = Graphics[{{PointSize[Large], Red, Point[pos1]}, Text[Style["+", 20, Bold, Black], pos1 + {0, 0.15}],  
{PointSize[Large], Blue, Point[pos2]}, Text[Style["-", 24, Bold, Black], pos2 + {0, 0.15}]}];
```

```
Show[equipotentials, fieldLines, chargeMarkers, PlotRange → {{-range, range}, {-range, range}}, AspectRatio → 1, Frame → True,  
FrameLabel → {"x", "y"}, PlotLabel → "Equipotentials (blue) and Field Lines (red)\nfor a Dipole (+1 at (1,0), -1 at (-1,0))",  
ImageSize → 500]
```